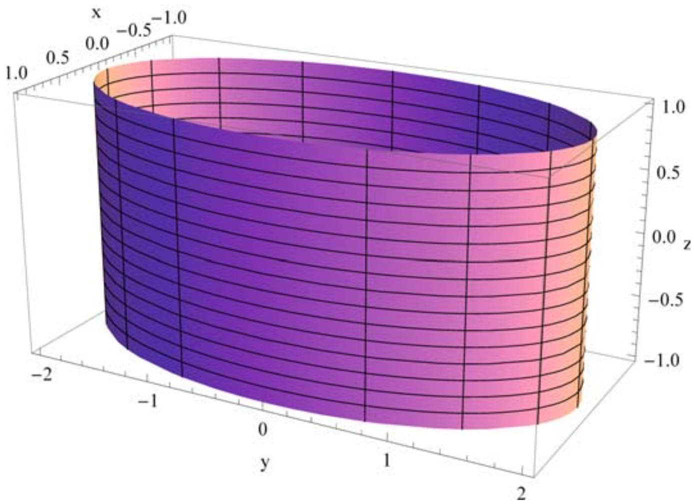
Parametric surface

```
Needs["Graphics`ParametricPlot3D`"];
```

Given the parametric surface $\boldsymbol{r}(u, v) = \cos u\ \boldsymbol{i} - 2\sin u\ \boldsymbol{j} + v\,\boldsymbol{k}$, $0 \le u \le 2\pi$, $-1 \le v \le 1$
Represent its surface and determine its area.

```
f1 = ParametricPlot3D[{Cos[u], -2 Sin[u], v}, {u, 0, 2 π},
  {v, -1, 1}, AxesLabel → {"x", "y", "z"}, ViewPoint → {2.4`, 1.3`, 1}]
```




```
r1[u_, v_] = {Cos[u], -2 Sin[u], v};
x1 = Cross[D[r1[u, v], u], D[r1[u, v], v]]
Simplify[Sqrt[x1.x1]]
```

{-2 Cos[u], Sin[u], 0}

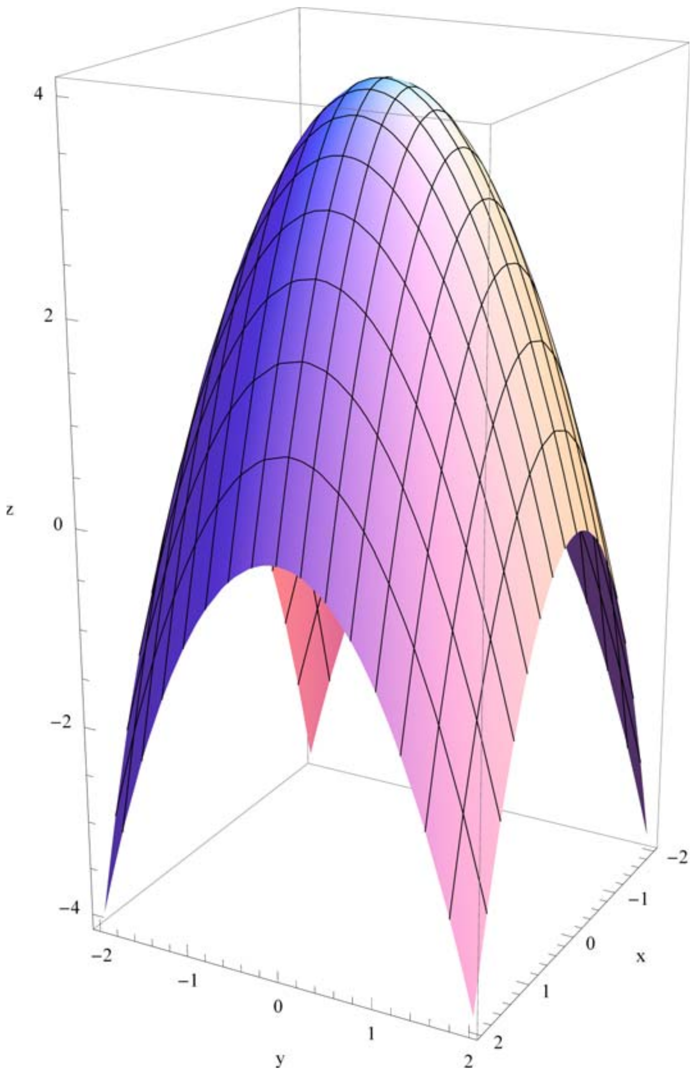
$\sqrt{4\,\text{Cos}[u]^2 + \text{Sin}[u]^2}$

```
As1 = Integrate[Integrate[Sqrt[x1.x1], {u, 0, 2 π}], {v, -1, 1}]
```

$16\,\text{EllipticE}\left[\frac{3}{4}\right]$

Given the parametric surface $\boldsymbol{r}(u, v) = u\ \boldsymbol{i} + v\ \boldsymbol{j} + \left(4 - u^2 - v^2\right)\boldsymbol{k}$**,** $-2 \le u \le 2,\ -2 \le v \le 2$
Represent its surface and determine its area.

```
f2 = ParametricPlot3D[{u, v, 4 - u^2 - v^2}, {u, -2, 2}, {v, -2, 2},
  BoxRatios → {1, 1, 2}, AxesLabel → {"x", "y", "z"}, ViewPoint → {2.4`, 1.3`, 1}]
```




```
r2[u_, v_] = {u , v, 4 - u^2 - v^2};
x2 = Cross[D[r2[u, v], u], D[r2[u, v], v]]
Simplify[Sqrt[x2.x2]]
```

$\{2u, 2v, 1\}$

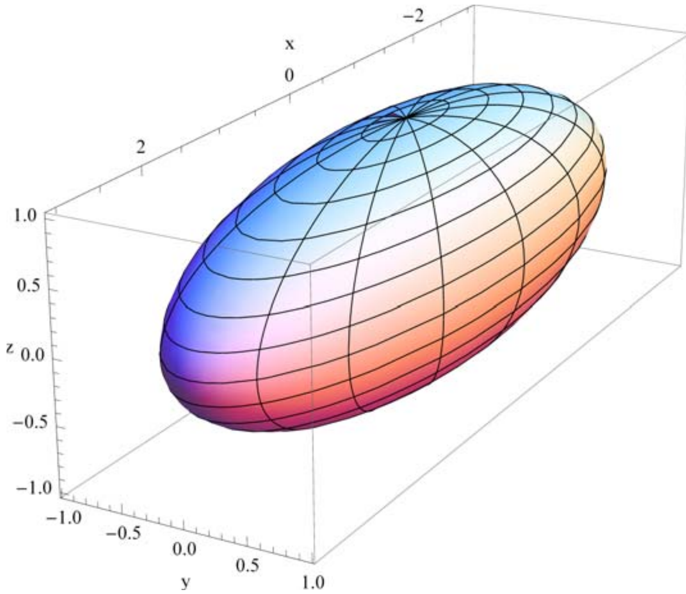
$\sqrt{1 + 4u^2 + 4v^2}$

```
As2 = Integrate[Integrate[Sqrt[x2.x2], {u, -2, 2}], {v, -2, 2}]
```

$$\frac{1}{3}\left(16\sqrt{33} + 38\,\mathrm{ArcSinh}\left[\frac{4}{\sqrt{17}}\right] - \mathrm{ArcTan}\left[\frac{16}{\sqrt{33}}\right] + 38\,\mathrm{ArcTanh}\left[\frac{4}{\sqrt{33}}\right]\right)$$

Given the parametric surface
$\boldsymbol{r}(u, v) = 3 \sin u \cos v\, \boldsymbol{i} + \sin u \sin v\, \boldsymbol{j} + \cos u\, \boldsymbol{k}$, $0 \le u \le \pi$, $0 \le v \le 2\pi$
Represent its surface and determine its area.

```
f3 = ParametricPlot3D[{3 Sin[u] Cos[v], Sin[u] Sin[v], Cos[u]},
  {u, 0, π}, {v, 0, 2 π}, AxesLabel → {"x", "y", "z"}, ViewPoint → {2.4`, 1.3`, 1}]
```




```
r3[u_, v_] = {3 Sin[u] Cos[v], Sin[u] Sin[v], Cos[u]};
x3 = Cross[D[r3[u, v], u], D[r3[u, v], v]]
Simplify[Sqrt[x3.x3]]
```

$\{\text{Cos}[v]\,\text{Sin}[u]^2,\ 3\,\text{Sin}[u]^2\,\text{Sin}[v],\ 3\,\text{Cos}[u]\,\text{Cos}[v]^2\,\text{Sin}[u] + 3\,\text{Cos}[u]\,\text{Sin}[u]\,\text{Sin}[v]^2\}$

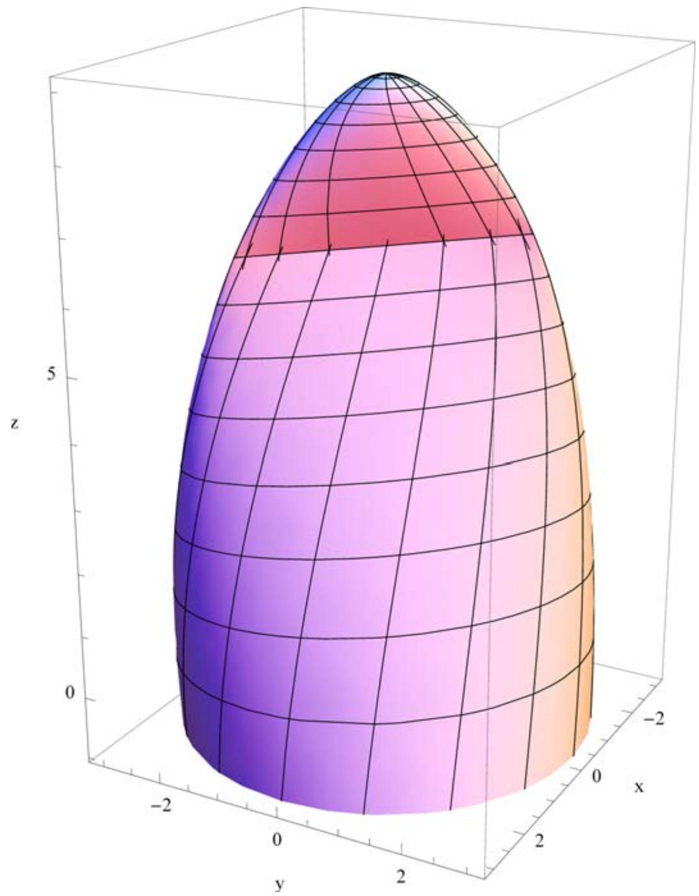
$\sqrt{(7 + 2\,\text{Cos}[2\,u] + \text{Cos}[2\,(u - v)] - 2\,\text{Cos}[2\,v] + \text{Cos}[2\,(u + v)])\,\text{Sin}[u]^2}$

```
As3 = Integrate[Integrate[Sqrt[x3.x3], {u, 0, π}], {v, 0, 2 * π}]
```

```
∫_0^(2π) If[ (8 Sec[v]^2 + Tan[v]^2 ∉ Reals || 4 + Re[(-7 + 2 Cos[2 v]) Sec[v]^2] ≤ 0 ||
      (Re[(-7 + 2 Cos[2 v]) Sec[v]^2] ≥ 4 && 1 + Re[8 Sec[v]^2 + Tan[v]^2] ≤ 0)) && 4 Cos[2 v] ≠ 5,
   -1/8 (-6 - 5 Sqrt[1 + Cos[2 v]] Log[4] + 5 Sqrt[1 + Cos[2 v]] Log[8] - 5/2 Sqrt[1 + Cos[2 v]] Log[2 Cos[v]^2] +
       5 Sqrt[1 + Cos[2 v]] Log[-2 - 2 Cos[2 v] + 3 Sqrt[1 + Cos[2 v]]] +
       2 Cos[2 v] (-3 + 2 Sqrt[1 + Cos[2 v]] Log[4] - 2 Sqrt[1 + Cos[2 v]] Log[8] +
          Sqrt[1 + Cos[2 v]] Log[2 Cos[v]^2] - 2 Sqrt[1 + Cos[2 v]] Log[-2 - 2 Cos[2 v] + 3 Sqrt[1 + Cos[2 v]]]))
     Sec[v]^2 - 1/16 (-12 - 5 Sqrt[1 + Cos[2 v]] Log[2] + 5 Sqrt[1 + Cos[2 v]] Log[Cos[v]^2] -
       10 Sqrt[1 + Cos[2 v]] Log[2 + 2 Cos[2 v] + 3 Sqrt[1 + Cos[2 v]]] +
       4 Cos[2 v] (-3 + Sqrt[1 + Cos[2 v]] Log[2] - Sqrt[1 + Cos[2 v]] Log[Cos[v]^2] +
          2 Sqrt[1 + Cos[2 v]] Log[2 + 2 Cos[2 v] + 3 Sqrt[1 + Cos[2 v]]])) Sec[v]^2,
   Integrate[Sqrt[7 + 2 Cos[2 u] + Cos[2 (u - v)] - 2 Cos[2 v] + Cos[2 (u + v)]] Sin[u],
    {u, 0, π}, Assumptions →
     ! ((8 Sec[v]^2 + Tan[v]^2 ∉ Reals || 4 + Re[(-7 + 2 Cos[2 v]) Sec[v]^2] ≤ 0 || (Re[(-7 + 2 Cos[2 v])
                Sec[v]^2] ≥ 4 && 1 + Re[8 Sec[v]^2 + Tan[v]^2] ≤ 0)) && 4 Cos[2 v] ≠ 5)]] dv
```

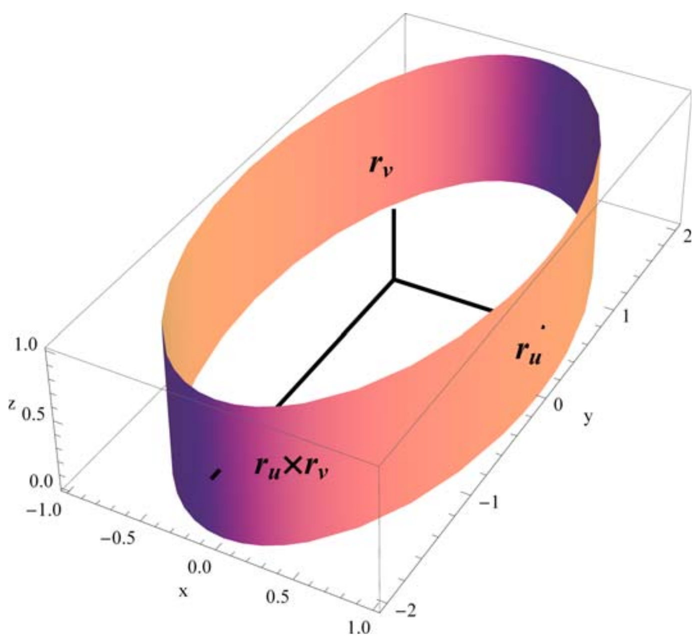
Given the parametric surface $\boldsymbol{r}(u, v) = u\cos(u+v)\,\boldsymbol{i} + u\sin v\,\boldsymbol{j} + (9-u^2)\,\boldsymbol{k}$, $0 \le u \le 3,\ 0 \le v \le 2\pi$
Represent its surface and determine its area.

```
h1 = ParametricPlot3D[{u Cos[u + v], u Sin[v], 9 - u^2}, {u, 0, π},
  {v, 0, 2 π}, AxesLabel → {"x", "y", "z"}, ViewPoint → {2.4`, 1.3`, 1}]
```



Given the parametric surface $\mathbf{r}(u, v) = 2\cos u\ \mathbf{i} + 2\sin u\ \mathbf{j} + v\,\mathbf{k}$, $0 \le u \le 2\pi$, $0 \le v \le 1$ and the vectors $\mathbf{r}_u$, $\mathbf{r}_v$, and $\mathbf{r}_u \times \mathbf{r}_v$ in (u,v)=(0,1/2)
Represent its surface ,given vectors , and determine its area.

```
g1 = ParametricPlot3D[{Cos[u], -2 Sin[u], v},
   {u, 0, 2 π}, {v, 0, 1}, AxesLabel → {"x", "y", "z"}, Mesh → None];
g3 = Graphics3D[{AbsoluteThickness[2], Line[{{0, 0, 0.5`}, {0, 0, 1.5}}]}];
g4 = Graphics3D[{AbsoluteThickness[2], Line[{{0, 0, 0.5`}, {1, 0, .5`}}]}];
g5 = Graphics3D[{AbsoluteThickness[2], Line[{{0, 0, 0.5`}, {0, -2.5, 0.5`}}]}];
g6 = Graphics3D[
   {Text[Style["\!\(\*SubscriptBox[\(r\), \(u\)]\)", FontSize → 16, FontWeight → "Bold"],
     {1, 0, 0.5}, {1, 1}], Text[Style["\!\(\*SubscriptBox[\(r\), \(v\)]\)",
      FontSize → 16, FontWeight → "Bold"], {0, 0, 1.5`}, {1, 1}],
    Text[Style["\!\(\*SubscriptBox[\(r\), \(u\)]\)×\!\(\*SubscriptBox[\(r\), \(v\)]\) ",
      FontSize → 16, FontWeight → "Bold"], {0.75, -2, 1}, {1, 1}]}];
Show[
 g1,
 g3,
 g4,
 g5,
 g6]
```



```
r4[u_, v_] = {u Cos[v], u Sin[v], v};
x4 = Cross[D[r4[u, v], u], D[r4[u, v], v]]
Simplify[Sqrt[x4.x4]]
Integrate[Integrate[Sqrt[x4.x4], {u, 0, 2 * π}], {v, 0, 1}]
```

$\{\text{Sin}[v], -\text{Cos}[v], u\,\text{Cos}[v]^2 + u\,\text{Sin}[v]^2\}$

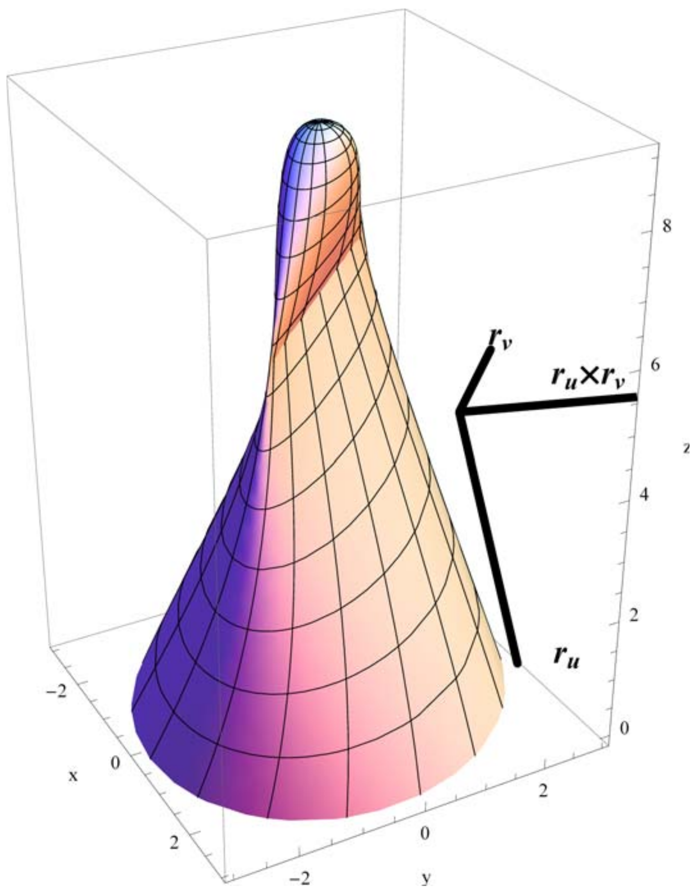
$\sqrt{1 + u^2}$

$\pi \sqrt{1 + 4 \pi^2} + \frac{1}{2} \text{ArcSinh}[2 \pi]$

Given the parametric surface $\boldsymbol{r}(u, v) = u\cos v\,\boldsymbol{i} + u\sin v\,\boldsymbol{j} + (9 - u^2)\,\boldsymbol{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$ and the vectors $\boldsymbol{r}_u$, $\boldsymbol{r}_v$, and $\boldsymbol{r}_u \times \boldsymbol{r}_v$ in $(u,v)=(2, \pi/4)$

Represent its surface ,given vectors , and determine its area.

```
h2 = ParametricPlot3D[{u Cos[u + v], u Sin[v], 9 - u^2}, {u, 0, 3},
   {v, 0, 2 π}, AxesLabel → {"x", "y", "z"}, ViewPoint → {2.4`, 1.3`, 1}];
h3 = Graphics3D[{AbsoluteThickness[4], Line[{{√2, √2, 5}, {3 √2/2, 3 √2/2, 1}}]}];
h4 = Graphics3D[{AbsoluteThickness[4], Line[{{√2, √2, 5}, {0, 2 √2, 5}}]}];
h5 = Graphics3D[{AbsoluteThickness[4], Line[{{√2, √2, 5}, {5 √2, 5 √2, 7}}]}];
h6 = Graphics3D[
   {Text[Style["\!\(\*SubscriptBox[\(r\), \(u\)]\)", FontSize → 16, FontWeight → "Bold"],
     {3 √2/2, 1.1` + 3 √2/2, 1.2`}, {1, 1}], Text[Style["\!\(\*SubscriptBox[\(r\), \(v\)]\)",
      FontSize → 16, FontWeight → "Bold"], {-0.5`, 2 √2 + 0.7`, 5.2`}, {1, 1}],
    Text[Style["\!\(\*SubscriptBox[\(r\), \(u\)]\)×\!\(\*SubscriptBox[\(r\), \(v\)]\) ",
      FontSize → 16, FontWeight → "Bold"], {3 √2, 3 √2 / 2, 7`}, {1, 1}]}];
Show[h2, h3, h4, h5, h6, ViewPoint → {2.4`, -1.3`, 1.5`}]
```




```
r5[u_, v_] = {u Cos[v], u Sin[v], 9 - u^2};
x5 = Cross[D[r5[u, v], u], D[r5[u, v], v]]
Simplify[Sqrt[x5.x5]]
Integrate[Integrate[Sqrt[x5.x5], {u, 0, 3}], {v, 0, 2 π}]
```

$\{2u^2 \mathrm{Cos}[v], 2u^2 \mathrm{Sin}[v], u \mathrm{Cos}[v]^2 + u \mathrm{Sin}[v]^2\}$

$\sqrt{u^2 + 4u^4}$

$\frac{1}{6}\left(-1 + 37\sqrt{37}\right)\pi$